

## 1. Piecewise Linear Approximation

HYENA is a computer model for system studies of Electrical Power Systems where the Production Capacity is mainly Hydro and Geothermal. HYENA uses a combination of Simulation and Optimization techniques. HYENA has been developed for Landsvirkjun, the National Power Company in Iceland.

In [1] the contribution of reservoir contents in the objective function, the future cost, is represented by:

$$\text{Min} \left\{ - \sum_k \left[ \int_0^{V_{k,t+1}} \rho_k(x) \cdot \alpha_{s,t+1}(x) \cdot dx \right] \right\} \quad (1)$$

where  $\rho_k(x)$  is the production coefficient of the  $k^{\text{th}}$  reservoir and  $\alpha_{s,t+1}(x)$  the water value of the  $s^{\text{th}}$  water area.  $V_{k,t+1}$  is the stored volume in reservoir  $k$  at end of time step  $t$ .

In the linear case  $\rho_k(x)$  and  $\alpha_{s,t+1}(x)$  are taken out of the integral and (1) is simplified to:

$$\text{Min} \left\{ - \sum_k \rho_k \cdot \alpha_{s,t+1} \cdot V_{k,t+1} \right\} \quad (2)$$

In (2)  $\rho_k$  and  $\alpha_{s,t+1}$  are estimated for every time step in the simulation.

Using the nonlinear method (1) requires enormous computer time compared to the linear method (2). Using the linear method (2), the solution tends to “swing up and down” between time steps which is inconvenient and unrealistic especially for operation of the smaller reservoirs. This has been a long lasting problem in Icelandic hydro power models.

In this report we use a piecewise linear approximation of (1) by introducing new variables  $\Delta V_{k,j,t+1}$ :

$$\text{Min} \left\{ - \sum_k 0 \cdot V_{k,t+1} - \sum_k \left[ \sum_{j=1}^{\text{Segments}} \chi_{k,j} \cdot \Delta V_{k,j,t+1} \right] \right\} \quad (3)$$

With the additional constraints:

$$\sum_{j=1}^{Segments} \Delta v_{k,j,t+1} = v_{k,t+1} \quad (4)$$

$$0 \leq \Delta v_{k,j,t+1} \leq \frac{v_{k,t+1}^{\max}}{Segments} \quad (5)$$

$\chi_{k,j}$  is estimated numerically according to (1).

The variable  $v_{k,t+1}$  is kept in the model to be consistent with former results and to simplify programming work.

## 2. Numerical Results

Figure 1 shows the  $\chi_{k,j}$  in one time step of a simulation run for 7 reservoirs.

Figure 1

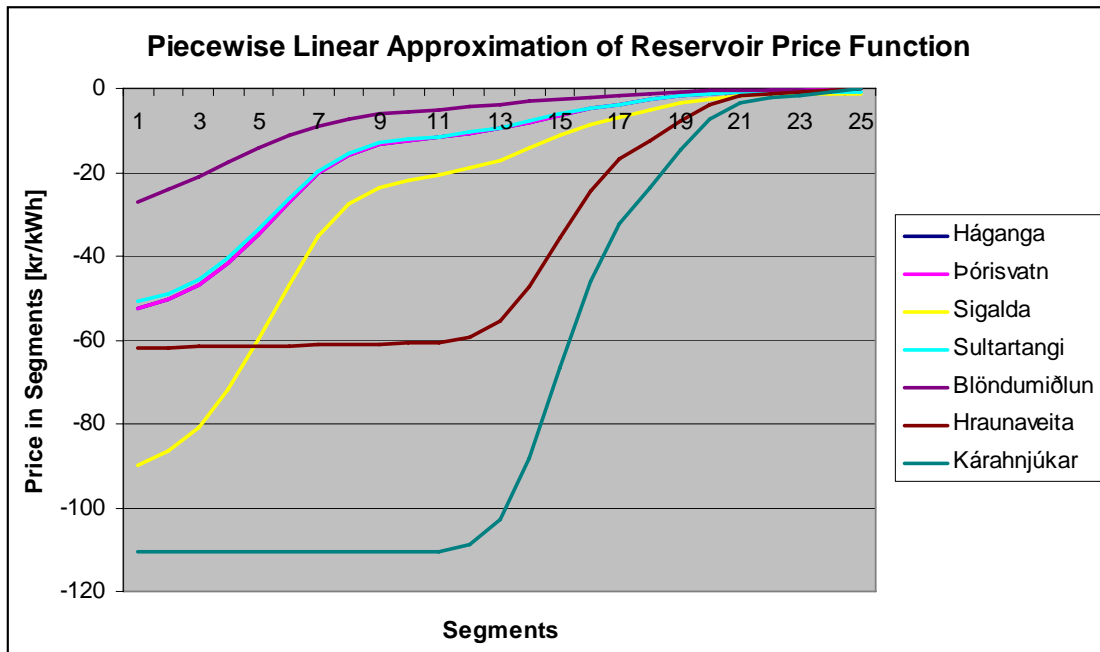
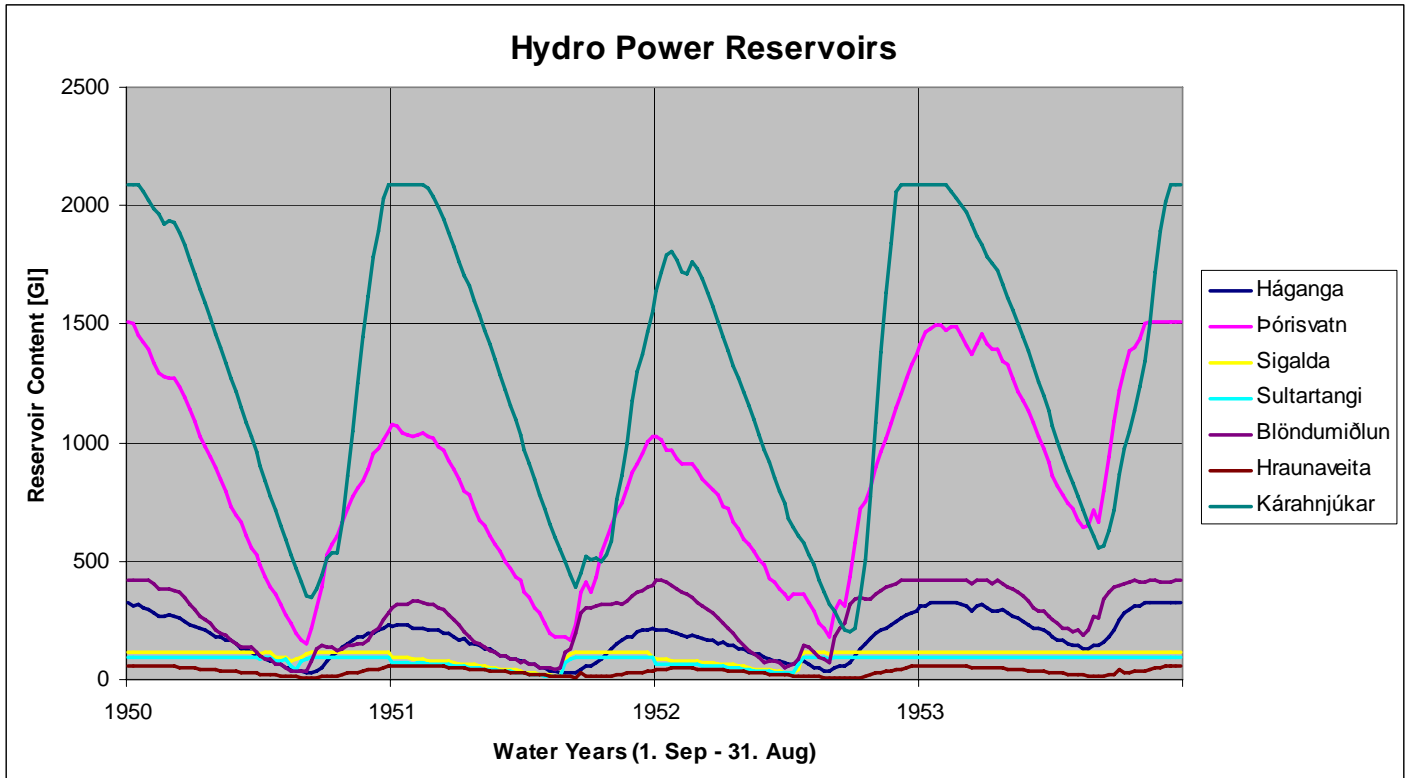


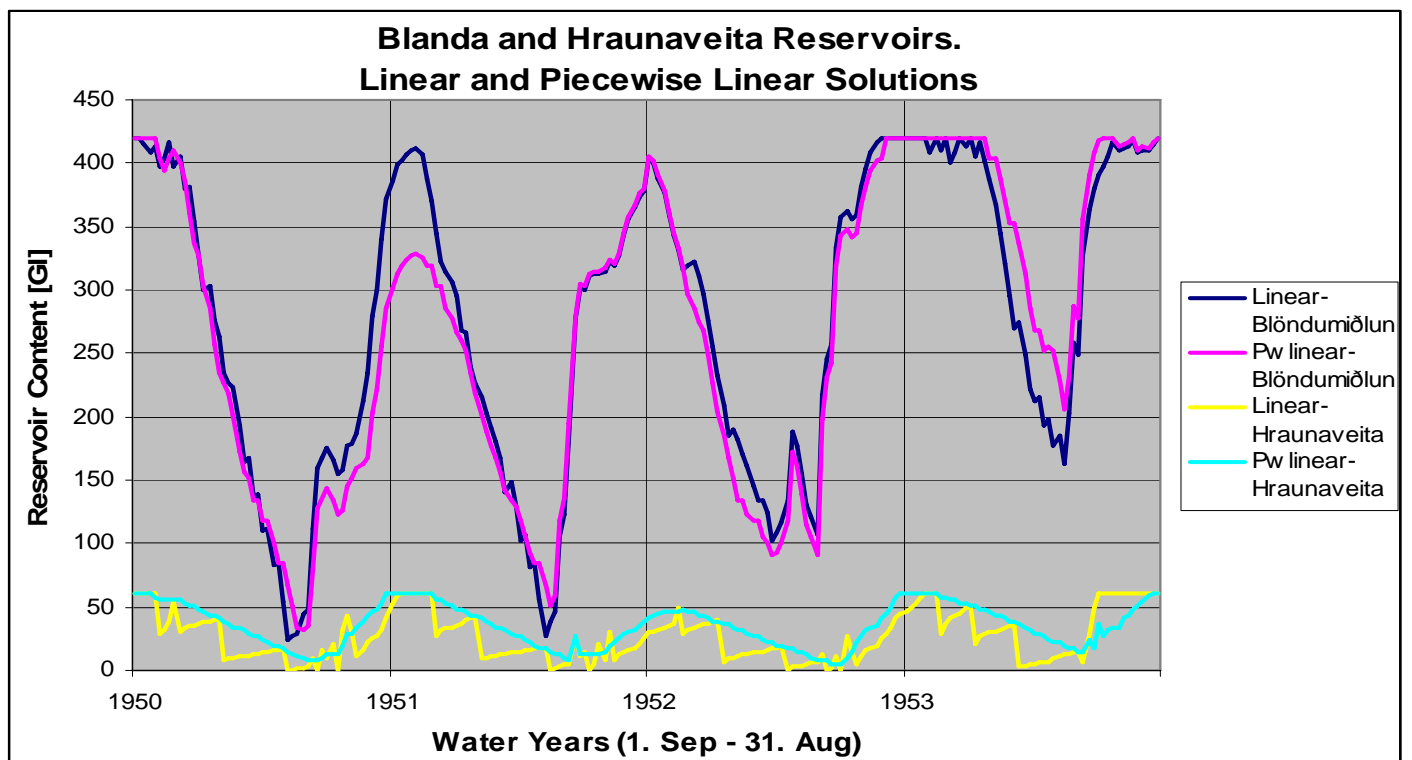
Figure 2 shows results of the simulation for 4 water years. In figure 1 Háganga reservoir curve has the same values as Þórisvatn and therefore hidden beneath.

Figure 2



The results in Figure 2 are much more streamlined than former results using the linear model (2) but it requires around twice as much computer time, with 25 segments in the piecewise linear approximation.

Figure 3



In figure 3 we compare the linear and piecewise linear solution for two reservoirs, the medium sized Blanda reservoir 420 Gl, and the smallest reservoir in the model, Hraunaveita 60 Gl.

The nonlinear case (1) is still almost impossible to use in practice because of excessive computer time. It is nevertheless valuable for comparison in development of the linear model as the underlying mathematical models are entirely different.

### 3. References

[1] HYENA (HYdro ENergy simulAtor). Some Issues in a Simulation Model of a Hydro Thermal Power System. Skuli Johannsson, Annad veldi ehf, Reykjavik, Iceland [skuli@veldi.is](mailto:skuli@veldi.is) and Elias B Eliasson The National Power Company, Iceland , [elias@lv.is](mailto:elias@lv.is) . 01-April-2005. <http://www.veldi.is/reports>

[2] Elias B Eliasson and Ulfar Linnet, both from Landsvirkjun The National Power Company, contributed to the report.

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